Optical Images (Optische Abbildungen)

OPA

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Introduction

In the practical experiment „Optical images“ the basics of geometrical optics are experimentally proved. The geometrical optics depicts the correlation of light and objects with essentially bigger dimensions than the wavelength of light, so the wave character of light can be ignored. That means that effects of refraction are irrelevant.

The following assumptions are made in geometrical optics:
- Light can be described by separate beams. One beam describes a direct line in a homogeneous medium.
- Different rays may intersect but aren’t influenced by each other.
- On the interface between two media with different speeds of light, the direction of a beam may change.

In addition the principal of Fermat applies:
Light covers the distance between two points by travelling (with regard to time) the shortest way. Because of different speeds of light in different media, the way that can be travelled in the shortest time, is not necessarily the shortest distance.

Experimental Setting

Optical components used:

− optical bench (with a scale from 1 cm to 140 cm)

− lenses: A, B, C, D, E, G, H (all lenses can be considered in a sufficient approximation as thin lenses)

− lights source (halogen lamp)

− objects to be projected:

  − slide with grid

  − slide with a small drawing of a car

− clips

− white screen

Exercise 5.1

Which of the lenses A, B, C, D, E, G H are condensor lenses, which are dispersion lenses?

Setup

The light source is clipped to the optical bench. About 100 $\pm$ 1 mm apart, we put a slide (small drawing of a car) and the lens to be examined.

Implementation

To determine which of the lenses where condensor lenses or dispersion lenses, we took a white screen and moved it away from the lens beginning from the nearest point possible. By examining the projection of the car and how it changed, we could decide on the type of lens:

If we found a focal point and the picture of the car was vertically inverted behind this point, the type was a condensor lens.

On the other hand, if the picture was just magnified (as a side−effect, it was actually also blurred), the type was a dispersion lens.

Test results

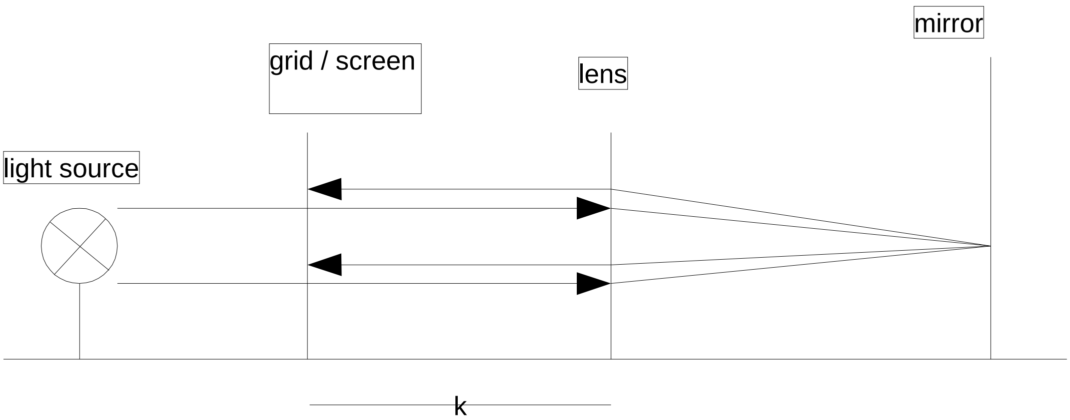
	A	B	C	D	E	G	H
condensor lens	x	x	x	x		x	x
dispersion lens					x		

Exercise 5.2

Determine the focal distances of lens B, C and D by means of autocollimation.

Setup

To determine the focal distance of the lenses to be tested, components are clipped to the optical bench as in the drawing below:



The distance between the light source and the grid / screen is about 100 $\pm$ 1 mm. The frame „grid / screen“ frame is half and half filled with the grid and the screen. We need to project the grid on itself to compare the size and sharpness of the picture and the projection. This could be done by turning the mirror a little, so the projection could be seen on the screen. Without the screen it would be very difficult / impossible to see the projection. The distance between grid / screen and the lens is k, the mirror is clipped to the optical bench about 150mm apart from the lens.

Implementation

Now we moved the lens to be tested between the grid/screen and the mirror until the projection was sharp and had the exact size as the object. We repeated each measurement 3 times to increase accuracy.

Test results

lens B	lens C	lens D
k_1 = 94,0 $\pm$ 1 mm	k_1 = 200,1 $\pm$ 1 mm	k_1 = 469,9 $\pm$ 1 mm
k_2 = 94,0 $\pm$ 1 mm	k_2 = 200,0 $\pm$ 1 mm	k_2 = 470,2 $\pm$ 1 mm
k_3 = 95,0 $\pm$ 1 mm	k_3 = 200,0 $\pm$ 1 mm	k_3 = 470,0 $\pm$ 1 mm
$k_{ø}$ = 94,0 $\pm$ 1 mm	$k_{ø}$ = 200,0 $\pm$ 1 mm	$k_{ø}$ = 470,0 $\pm$ 1 mm
$\Rightarrow$ f = 94,0 $\pm$ 1 mm	$\Rightarrow$ f = 200,0 $\pm$ 1 mm	$\Rightarrow$ f = 470,0 $\pm$ 1 mm

The given inaccuracy of the values is due to errors by reading the values from the scale.

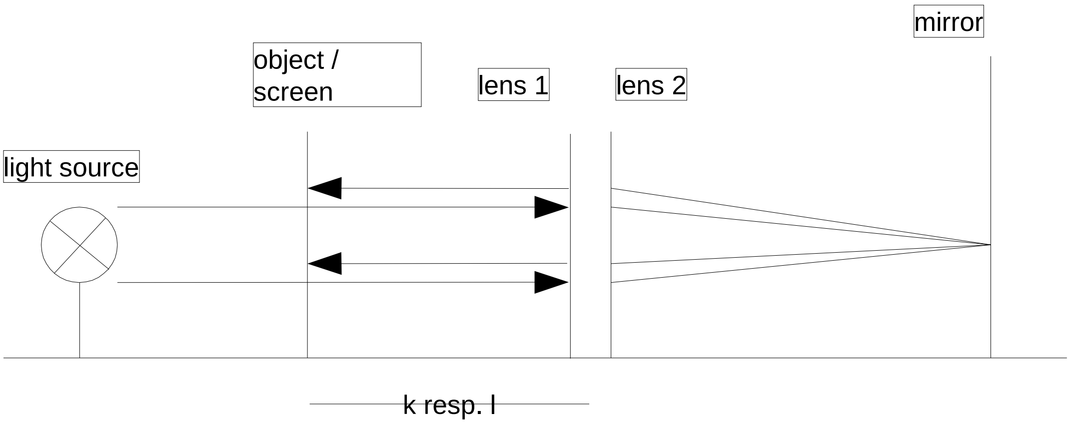
Exercise 5.3

Construct a system of lenses consisting of lens B and E. The distance between the lenses should amount to 40mm. Determine the focal distance and the Principal plane distance for the system. Use the...

Exercise 5.3.1 ...method of autocollimation and Bessel

Setup − method of autocollimation

As in the previous experiment, the method of autocollimation is used. But this time we use 2 lenses, in other words a system of lenses. The distance between lens 1 and lens 2 is 40 $\pm$ 1 mm. The distance between the system and the mirror amounts to about 200mm. As described in the previous setup, the frame containing the object/screen is half and half filled with the object (this time an image of a small car) and the white screen. The light source is about 100mm apart from the object/screen frame.



Implementation – method of autocollimation

During the first three measurements, lens 1 is our lens A and lens 2 is our lens E. The system of lenses is again moved until we see on the screen a sharp and in size identical projection of the object. The measurement is repeated 3 times, though it should be noted that we completely removed and replaced the system of lenses from the optical bench to get more objective results.

Now we turn the system by 180°, so that lens 1 becomes lens E and lens 2 becomes lens B. 3 new measurements are carried out after the identical pattern as before.

The values we measure are k in lens arrangement 1 (B – E) and l in arrangement 2 (E – B). The length of k and l are always from the object/screen to the middle of the system of lenses (we picked of the values from the scale exactly between lens 1 and lens 2).

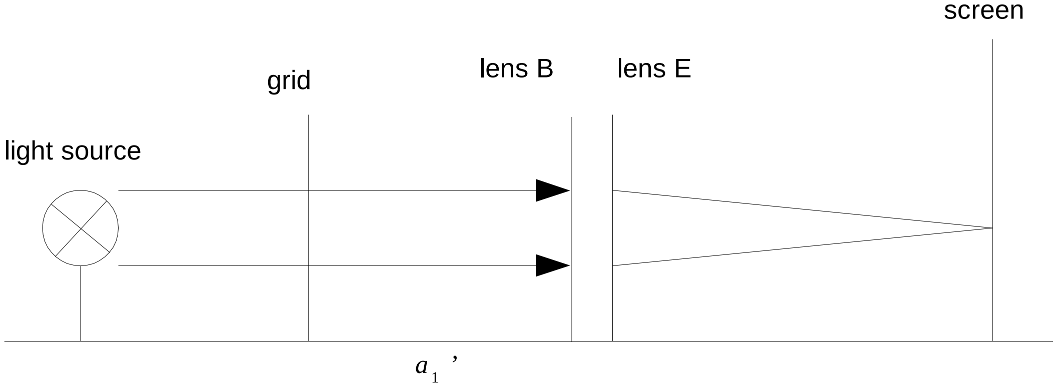
Test results – method of autocollimation

lens arrangement 1 (B − E)	**lens arrangement 2 (E − B)**
k_1 = 160,0 $\pm$ 1 mm	l_1 = 351,0 $\pm$ 1mm
k_2 = 159,9 $\pm$ 1 mm	l_2 = 351,0 $\pm$ 1mm
k_3 = 160,0 $\pm$ 1 mm	l_3 = 352,0 $\pm$ 1mm
$k_ø$ = 160,0 $\pm$ 1 mm	$l_ø$ = 351,0 $\pm$ 1mm
$\Rightarrow$ k = 160,0 $\pm$ 1 mm	$\Rightarrow$ l = 351,0 $\pm$ 1mm

The given inaccuracy of the values is due to errors by reading the values from the scale.

Setup – method of Bessel

With the Bessel method, the object (in this case the slide with the grid) is projected to the screen through lens B and E. The screen is clipped to the opical bench in a distance of e = 1100 $\pm$ 1 mm apart from the object (grid).



Implementation – method of Bessel

In the Bessel method, it is assumed that – given a large enough and fixed distance between the object (grid) and the screen – there exist 2 lens positions, where a sharp and real projection of the object be made. To find these two positions, we move the system of lenses as close as possible to the object (grid). Now we move it slowly in the direction of the screen until a shar projection can be seen on the screen. We take down the value a_1’ between the object (grid) and the middle of the system of lenses. As we move the system further, the projection gets blurred again until we get to the second position, where a sharp projection can be seen again. Once more, we take down the value of a_1’ . We repeat the whole procedure 3 times, initial position for the system of lenses is always the – due to technical limitations − closest distance to the object (grid).

Test results – method of Bessel

point 1	**point 2**
$a_1{'}_1$ = 289,9 ± 1 mm	$a_1{'}_1$ = 620,3 ± 1 mm
$a_1{'}_2$ = 290,0 ± 1 mm	$a_1{'}_2$ = 619,8 ± 1 mm
$a_1{'}_3$ = 290,2 ± 1 mm	$a_1{'}_3$ = 620,1 ± 1 mm
$a_1{'}_\varnothing$ = 290,0 ± 1 mm	$a_1{'}_\varnothing$ = 620,1 ± 1 mm

The given inaccuracy of the values is due to errors by reading the values from the scale.

⇒ d = 620,1 mm − 290,0 mm = 330,1 mm

Summary

With equation (18) in the experiment instructions we can calculate the focal distance of the system of lenses:

e = 1100mm

k = 160,0mm

l = 351,0mm

d = 330,1mm

$$f' = \frac{1}{2}\sqrt{(e-k-l)^2 - d^2} = 243{,}9 \text{ mm}$$

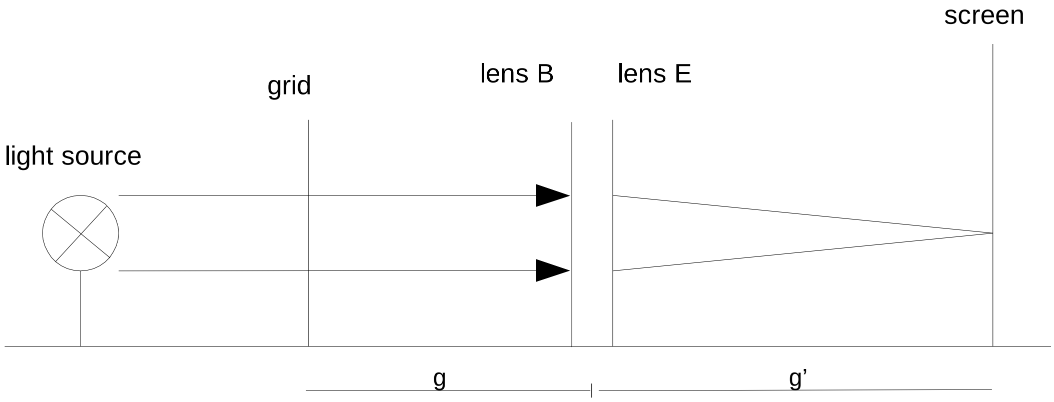
$$\Rightarrow f = -243{,}9 \text{ mm}$$

The distance of the principal plane is

$$h = k + l - 2f = (160{,}0 + 351{,}0 - 2 \cdot 243{,}9) \text{ mm} = 23{,}2 \text{ mm}$$

Exercise 5.3.2 Method of Abbé (graphically)

Setup



Implementation

At the beginning we set the distance between screen and slide (this time the 0.5 cm grid) to $e_1 = 1400 \pm 1$mm. Because the given intervals in the experiment instructions by which we should reduce the distance e hardly showed any difference in the projection of the grid on the screen, we decided to reduce the distance in intervals of δe = 50 mm.

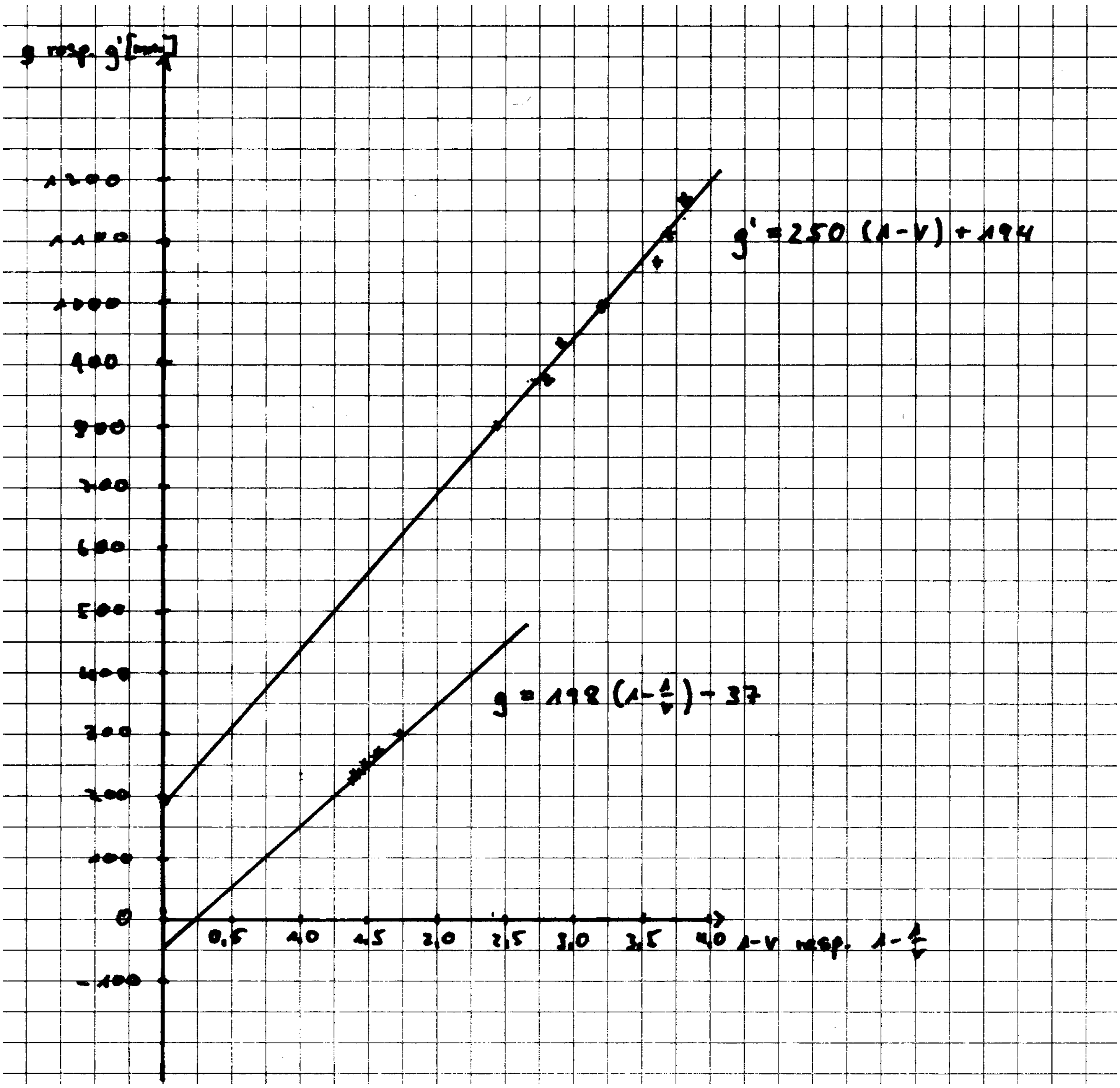
While reducing e, we moved the lens system into the position until we got a sharp projection of the grid. We took down the distance g (between the object (grid) and the middle of the system of lenses) and the distance g' (between the system and the screen) as well as the height of the grid squares on the screen y'.

Test results

Because the exact position of the system of lenses where a sharp projection could be seen was very difficult to determine (due to the fact that the image on the screen was quite contorted), we took as measured value the middle of the interval we believed the projection was sharp. This results to an inaccuracy of about 5 mm.

The magnigication factor is $V = \frac{y'}{y} = \frac{y'}{5\text{ mm}}$

	1	2	3	4	5	6	7
e [mm]	1400	1350	1300	1250	1200	1150	1100
g [mm]	230	235	235	255	265	275	300
g' [mm]	1170	1115	1065	995	935	875	800
y' [mm]	−14,0	−13,5	−13,0	−11,0	−9,5	−9,0	−7,0
V	−2,8	−2,7	−2,6	−2,2	−1,9	−1,8	−1,4



We calculated the linear fit with a mathematics program, because the error would even have been greater if we would have taken the values out of a graph.

The mathematics progam MAPLE V5.1 returns the following equation as linear fit:

$g = 198.08\ (1-1/V) - 36.74$

[program imput: with(stats): fit[leastsquare[[x,y]]]([[1.71,1.56,1.53,1.45,1.38,1.37,1.36],[300,275,265,255,235,235,230]]);

$g' = 250\ .00\ (1-V) + 193.57$

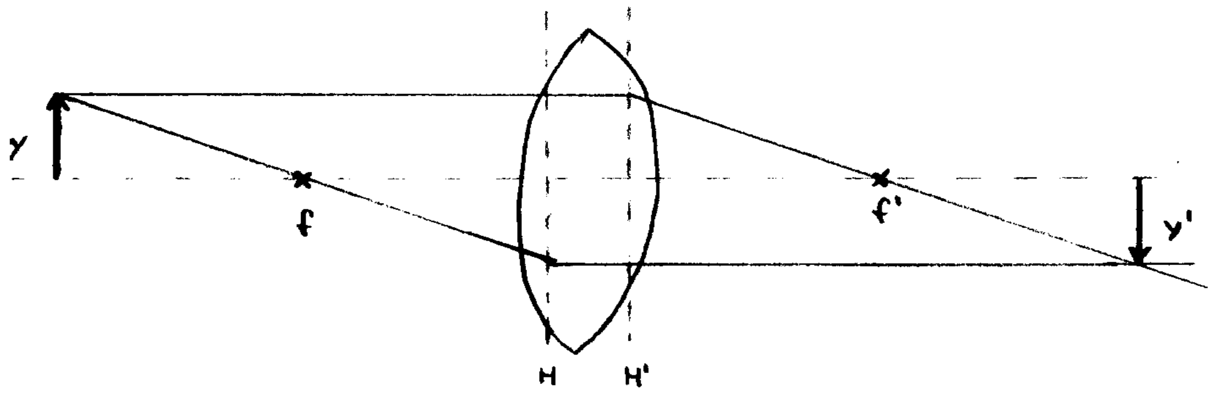
[program imput: with(stats):fit[leastsquare[[x,y]]]([[2.4,2.8,2.9,3.2,3.6,3.7,3.8],[800,875,935,995,1065,1115,1170]]);]

From the equation, we can pick the focal distance f and f' as well as the principal plane distances h_1 and h_2 of the system of lenses:

$f = 198$ mm, $f' = 250$ mm, $h = |h_1| + |h_2| = 194$ mm + 37 mm = 231 mm

We believe the inaccuracy of those values is so high, that error calculation would'n make sense. Furthermore, we did'n have the time to take more than one value for each position of the screen to calculate the statistical error.

Exercise 5.3.3 Determine graphically the image of an object at distance of 2f from the principal plane H.



Exercise 3.3.4 Calculate the focal distance of the system's dispersion lenses from the focal distances determined in 3.3.1 and 3.3.2 by means of equation (6) with t = 40 mm

$$f' = \frac{f_1' \; f_2'}{\Delta} = \frac{f_1' \; f_2'}{t - f_1' - f_2'}$$

$$f_1' \; f_2' = f't - f' \; f_1' - f' \; f_2'$$

$$f_2' = \frac{f' \; (t - f_1')}{f' + f_1'} = \frac{243{,}9\,(40 - 94)}{243{,}9 + 94} \text{ mm} = -39 \text{ mm}$$

Exercise 3.5 Slide Projector

For the slide projector, we used lenses A (as condensor) and C (as objective). We tried to move the optical bench with the components on as far as possible away from the wall where we projected the image, to get a large picture.

For details of the slide projector we built see answer to question 3.

Questions

1. What is a beam of light ?

A beam of light is a thin bunch of parallel light. In geometric optic you can assume, that different rays are independent of each other and reversible. It expands straightforward in three dimensional space in a homogeneous medium. The laws of refraction and reflection apply.

2. What is the proportion of the two pictures, created in the positions 1 and 2 ?

The scaling factors β_1 and β_2 are calculated as follows:

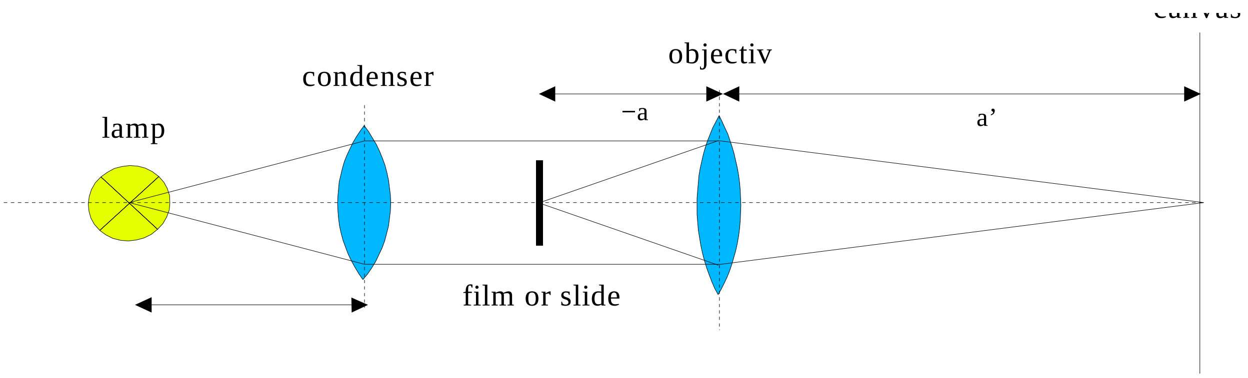
$$\beta_1 = \frac{y_1'}{y_1} \,(4) = \frac{a_1'}{a_1} = \frac{a'}{a} \quad \text{with} \quad \begin{array}{l} a = a_1 = -a_2' = (d + h - e)/2 \\ a' = a_1' = -a_2 = (d - h + e)/2 \end{array}$$

$$\beta_2 = \frac{y_2'}{y_2} \,(4) = \frac{a_2'}{a_2} = \frac{a}{a'} = \frac{1}{\beta_1}$$

The original size of the object is fix: $y_1 = y_2 = y$

$$\boxed{\frac{y_1'}{y} = \frac{y}{y_2'} \quad \Rightarrow \quad y_1' \cdot y_2' = y^2} \quad \Rightarrow \beta_1 = \frac{1}{\beta_2}$$

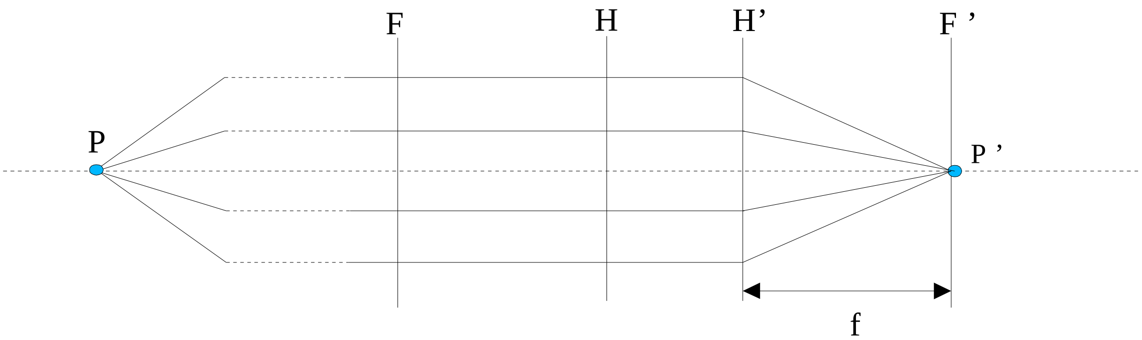
3. How are the lamp, the condenser and the objective arranged in a projector ?



The lamp should be in the absolute focus of the condenser lens to get parallel beams on the other side. For a and a’ the following equation applies:

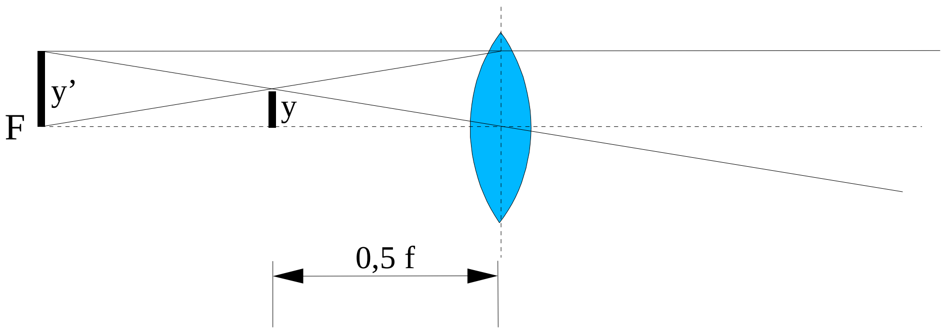
$$\frac{1}{a} - \frac{1}{a'} = \frac{1}{f}$$ where f is the focal distance of the objective.

4. How long is the distance between the film layer and the main layer on the picture side of a telephoto with a focal distance of 200 mm set to infinite ?



If you assume that P is indefinitely far away, all beams of light are parallel and focused in the focal layer in P’. So the distance between the main layer and the film should be the focal distance (200 mm), to get a sharp image.

5. What results from the equations (3a,b) respectively (4) for the position, the size and the nature of the image if an object resides in a distance of 0,5 f in front of a lens ?



$$a = 0{,}5 \cdot f$$

$$\boxed{\frac{1}{a}-\frac{1}{a'}=\frac{1}{f}\,(3)\quad\Rightarrow\quad\frac{2}{f}-\frac{1}{a'}=\frac{1}{f}\quad\Rightarrow\quad\frac{1}{f}=\frac{1}{a'}\quad\Rightarrow\quad f=a'}$$

Enlargement:

$$\boxed{\beta=\frac{y'}{y}\,(4)=\frac{a'}{a}=\frac{f}{0{,}5\ f}=2}$$

The image is virtual, that means it is on the same side of the lens as the original object. It has the double size of the object and the same orientation.

6. How changes the total focal distance (f') of a system consisting of two lenses with the same focal distance (f > 0) in relativity to the distance t (according to equation (6)) ?

$$f' = \frac{f^2}{\Delta}\,(6)$$

$$h=\frac{t^2}{\Delta}\,(8)\quad\Rightarrow\quad\Delta=\frac{t^2}{h}$$

$$\boxed{f' = \frac{f^2}{t^2}\cdot h}$$